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GIMP application
Subject: Math (III grade)
Topic: Equation of a circle

Creator: Nada Sirmevska
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#### Abstract

In plane geometry, a circle is defined as the set of all points which are equidistant from a fixed point. In the coordinate plane, a circle can be represented by an equation which is the relation between the points on the circle and the fixed elements, i.e. the center and the radius of the circle.


## Standard equation of a circle

Theorem: In the coordinate plane, the equation of a circle with center $\mathrm{C}(\mathrm{a}, \mathrm{b})$ and radius r is $(x-a)^{2}+(y-b)^{2}=r^{2}$.

Proof: Let $\mathrm{P}(\mathrm{x}, \mathrm{y})$ be any point on the circle, $C(a, b)$ be the center and $r$ be the radius of the circle. Since $r=C P$, by using the distance formula we can write
$C P=\sqrt{(x-a)^{2}+(y-b)^{2}}=r$. By taking the square of both sides of the equation, we get $(x-a)^{2}+(y-b)^{2}=r^{2}$. This is the standard equation of a circle.


Example 1: Write the equation of the circle with center $\mathrm{C}(-3,2)$ and radius $\mathrm{r}=5$.
Solution: $\mathrm{C}(\mathrm{a}, \mathrm{b})=\mathrm{C}(-3,2)$ and $\mathrm{r}=5$. The equation of the circle is $(x-(-3))^{2}+(y-2)^{2}=5^{2}$, i.e. $(x+3)^{2}+(y-2)^{2}=25$. This is the equation of the circle.


## Erasmus+

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Example 2; Find the center and radius of the circle $\left(x-\frac{2}{3}\right)^{2}+(y+1)^{2}=\frac{1}{4}$.

Solution: If we compare the equations
$(x-a)^{2}+(y-b)^{2}=r^{2}$ and $\left(x-\frac{2}{3}\right)^{2}+(y+1)^{2}=\frac{1}{4}$ we can see that $\mathrm{C}(\mathrm{a}, \mathrm{b})=\left(\frac{2}{3},-1\right)$ and $\mathrm{r}=\frac{1}{2}$.


## Remark:

1. The equation of a circle with center at the origin $O(0,0)$ and radius $r$ is $x^{2}+y^{2}=r^{2}$.
2. The equation of a circle with center on the x -axis and radius $r$ is
$(x-a)^{2}+y^{2}=r^{2}$.
3. The equation of a circle with center on the $y$-axis and radius $r$ is $x^{2}+(y-b)^{2}=r^{2}$.




Note: A circle whose center is at the origin is called a central circle.

## General equation of a circle

We have seen that the standard equation of a circle with center $C(a, b)$ and radius $r$ is

$$
(x-a)^{2}+(y-b)^{2}=r^{2} .
$$

By expanding this equation we obtain

$$
\begin{align*}
x^{2}-2 a x+a^{2}+y^{2}-2 b y+b^{2} & =r^{2} \\
x^{2}+y^{2}-2 a x-2 b y+a^{2}+b^{2}-r^{2} & =0 \tag{1}
\end{align*}
$$

Now, let us take $\mathrm{D}=-2 \mathrm{a}, \mathrm{E}=-2 \mathrm{~b}$, and $\mathrm{F}=a^{2}+b^{2}-r^{2}$ and substitute in (1).
We find $x^{2}+y^{2}+D x+E y+F=0$.
Definition: The equation $x^{2}+y^{2}+D x+E y+F=0$ where $\mathrm{D}=-2 \mathrm{a}, \mathrm{E}=-2 \mathrm{~b}$, and $\mathrm{F}=a^{2}+b^{2}-r^{2}$ is called the general equation of a circle.

Remark: The general equation of a circle gives the following properties.

1. $\left.\begin{array}{rl}D & =-2 a \Rightarrow a=-\frac{D}{2} \\ E & =-2 b \Rightarrow b=-\frac{E}{2}\end{array}\right\}$ the center $\mathrm{C}(\mathrm{a}, \mathrm{b})=\mathrm{C}\left(-\frac{D}{2},-\frac{E}{2}\right)$.
2. $\mathrm{F}=a^{2}+b^{2}-r^{2} \Longrightarrow \mathrm{r}=\sqrt{a^{2}+b^{2}-F}=\sqrt{\frac{D^{2}}{4}+\frac{E^{2}}{4}-F}$ so the radius

$$
\mathrm{r}=\frac{1}{2} \sqrt{D^{2}+E^{2}-4 F} .
$$

Example 3: Find the general equation of the circle with center $\mathrm{C}(-2,3)$ and radius $\mathrm{r}=5$.
Solution: The general equation of the circle is $x^{2}+y^{2}+D x+E y+F=0$ where $\mathrm{D}=-2 \mathrm{a}$, $\mathrm{E}=-2 \mathrm{~b}$, and $\mathrm{F}=a^{2}+b^{2}-r^{2}$, so $\mathrm{D}=-2 \cdot(-2)=4, \mathrm{E}=-2 \cdot 3=-6$ and $\mathrm{F}=(-2)^{2}+3^{2}-5^{2}=-12$. So the equation is $x^{2}+y^{2}+4 x-6 y-12=0$

