

**Erasmus** <u>http://ioerc.mk</u> **GIMP application** Subject: Math (III grade) Topic: Equation of a circle

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Subject: Math (III grade)

**Topic:** Equation of a circle

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**Abstract:** In plane geometry, a circle is defined as the set of all points which are equidistant from a fixed point. In the coordinate plane, a circle can be represented by an equation which is the relation between the points on the circle and the fixed elements, i.e. the center and the radius of the circle.

## Standard equation of a circle

<u>**Theorem:**</u> In the coordinate plane, the equation of a circle with center C(a,b) and radius r is  $(x-a)^2 + (y-b)^2 = r^2$ .

Proof: Let P(x,y) be any point on the circle, C(a,b) be the center and r be the radius of the circle. Since r=CP, by using the distance formula we can write

 $CP = \sqrt{(x-a)^2 + (y-b)^2} = r$ . By taking the square of both sides of the equation, we get  $(x-a)^2 + (y-b)^2 = r^2$ . This is the standard equation of a circle.



*Example 1:* Write the equation of the circle with center C(-3,2) and radius r=5.

Solution: C(a,b)=C(-3,2) and r=5. The equation of the circle is  $(x-(-3))^2 + (y-2)^2 = 5^2$ , i.e.  $(x+3)^2 + (y-2)^2 = 25$ . This is the equation of the circle.



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*Example 2:* Find the center and radius of the circle  $(x - \frac{2}{3})^2 + (y + 1)^2 = \frac{1}{4}$ .

Solution: If we compare the equations  $(x-a)^2 + (y-b)^2 = r^2$  and  $(x-\frac{2}{3})^2 + (y+1)^2 = \frac{1}{4}$  we can see that  $C(a,b) = (\frac{2}{3},-1)$  and  $r = \frac{1}{2}$ .

Remark:

1. The equation of a circle with center at the origin O(0,0) and radius r is  $x^2 + y^2 = r^2$ . 2. The equation of a circle with center on the x-axis and radius r is  $(x-a)^2 + y^2 = r^2$ . 3. The equation of a circle with center on the y-axis and radius r is  $x^{2} + (y-b)^{2} = r^{2}$ .



Note: A circle whose center is at the origin is called a central circle.





## General equation of a circle

We have seen that the standard equation of a circle with center C(a,b) and radius r is

$$(x-a)^2 + (y-b)^2 = r^2.$$

By expanding this equation we obtain

$$x^{2} - 2ax + a^{2} + y^{2} - 2by + b^{2} = r^{2}$$
$$x^{2} + y^{2} - 2ax - 2by + a^{2} + b^{2} - r^{2} = 0 \dots (1)$$

Now, let us take D=-2a, E=-2b, and F= $a^2 + b^2 - r^2$  and substitute in (1).

We find  $x^2 + y^2 + Dx + Ey + F = 0$ .

**Definition:** The equation  $x^2 + y^2 + Dx + Ey + F = 0$  where D=-2a, E=-2b, and  $F = a^2 + b^2 - r^2$  is called the general equation of a circle.

Remark: The general equation of a circle gives the following properties.

1. 
$$\begin{aligned}
D &= -2a \Rightarrow a = -\frac{D}{2} \\
E &= -2b \Rightarrow b = -\frac{E}{2}
\end{aligned}$$
 the center C(a,b)=C( $-\frac{D}{2}, -\frac{E}{2}$ ).  
2.  $F = a^2 + b^2 - r^2 \Rightarrow r = \sqrt{a^2 + b^2 - F} = \sqrt{\frac{D^2}{4} + \frac{E^2}{4} - F}$  so the radius  $r = \frac{1}{2}\sqrt{D^2 + E^2 - 4F}$ .

*Example 3:* Find the general equation of the circle with center C(-2,3) and radius r=5.

Solution: The general equation of the circle is  $x^2 + y^2 + Dx + Ey + F = 0$  where D=-2a, E=-2b, and F= $a^2 + b^2 - r^2$ , so D=-2·(-2)=4, E=-2·3=-6 and F=(-2)<sup>2</sup>+3<sup>2</sup>-5<sup>2</sup>=-12. So the equation is  $x^2 + y^2 + 4x - 6y - 12 = 0$